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A rejoinder to O’Donnell’s critique of the ergodic/nonergodic explanation of Keynes’s concept of uncertainty

Abstract: O’Donnell’s attack on the ergodic/nonergodic approach to Keynes’s uncertainty concept is erroneous. Keynes’s produced a general theory by imposing fewer restrictive axioms than classical theory. Keynes’s explicitly attacked the classical theory’s axiom that an actuarially certain knowledge regarding the future is available to decision makers; thus Keynes’s analysis rejects the ergodic hypothesis. Keynes’s theory does not substitute the assumption that all economic activity must be nonergodic. Keynes’s theory simply permits an analysis where the payout for some economic decisions (e.g., “animal spirits” investment spending) are made under uncertainty, i.e., without actuarial knowledge of the future payout of such spending decisions. O’Donnell’s epistemological approach implicitly supports the laissez faire view that there should not be any government interference with operation of free product and financial markets for government interference policies may only worsen future outcomes in a world where humans including Keynes, economists, and government officials do not have the capability to understand how the economy works.

Key words: ergodic, uncertainty, ontological, epistemological, probability distribution

The lead article in the winter 2014–15 issue of the Journal of Post Keynesian Economics, by Rod O’Donnell presents his criticism of “the ontologically oriented ergodic/nonergodic (ENE) approach

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advocated by Paul Davidson and his followers” (O’Donnell, 2014–15, p. 187). O’Donnell argues in favor of an “epistemologically oriented” belief in humans’ limited knowledge of the economy and the system that generates economic data that Keynes (1937, p. 151) called “facts.” In his “critique” O’Donnell claims that the ergodic vs. nonergodic approach to uncertainty is “untenable” and that “its treatment of Keynes’s... work is inaccurate and tendentious” (O’Donnell, 2014–15, p. 187).

We will show by explicit quotes from Keynes that it is O’Donnell’s treatment of Keynes’s work that is “inaccurate and tendentious.” Moreover his claims about the path that ergodic and nonergodic stochastic series can take before infinity indicate a lack of knowledge on his part of the concept of convergence to a limit in the calculus and the theory of stochastic processes.

Although I had introduced the ergodic/nonergodic approach to place Keynes’s concept of uncertainty in terms of modern theories of economics and stochastic probability theories, this is my first opportunity to provide a rejoinder to O’Donnell’s argument.

In his critique O’Donnell displays his lack of knowledge of (1) Keynes’s 1937 explicit indication that the General Theory rejects the twentieth-century classical theorists’ presumption that the calculus of probability analysis will provide an actuarially certain future of economic outcomes. The General Theory rejected this axiom in order to provide for an analysis of decision making where the future was uncertain. (2) The meaning of the concept of an axiom in the construction of a theory and Keynes’s claim that a general theory is one that requires fewer restrictive axioms. (3) The meaning of limits and conversion of stochastic processes. (4) The modern theory of stochastic processes (i.e., modern probability theory) in general.

Labeling a theory a general theory indicates that this theory has fewer restrictive axioms than the opposing theory. As I have noted previously (Davidson, 2007, pp. 26–35), Keynes’s General Theory removed three restrictive axioms that are fundamental to twentieth-century classical theory, namely (1) the neutral money axiom, (2) the gross substitution axiom, and (3) the ergodic axiom.

The ergodic axiom restricts the theory to economic systems where future outcomes can be known with actuarial certainty. There can be no uncertainty about actuarial future outcomes if decision makers are willing to calculate a probability distribution from existing data-facts. Removal of the restrictive ergodic axiom
implicitly used by twentieth-century classical theorists permitted Keynes to introduce his concept of uncertainty about the economic future into his *General Theory*.

The first error we should point out in O'Donnell’s critique involves his apparent lack of knowledge of Keynes’s indication of the twentieth-century classical axiom, which Keynes *explicitly* threw over to create a general theory that would encompass his concept of uncertainty. O’Donnell (2014–15. p. 207) states, “The basic axiom that Keynes abandoned in the *GT* was not the ergodic axiom but the perfect knowledge axiom.”

Accordingly, O’Donnell would apparently still permit the ergodic axiom to be compatible with Keynes’s analysis. Interestingly, the prime Keynesian of mainstream economic theory apparently agrees that the ergodic axiom was not, and must not, be abandoned in macroeconomic theory. Samuelson specifically states (1969, p. 112) that the ergodic axiom is essential in making economics a science.

Nevertheless, despite the fact that most of the mainstream economic establishment theorists have completely ignored Post Keynesian theory, three Nobel Prize winners in economics have recognized the importance of my Post Keynesian analysis associating Keynes’s concept of uncertainty with nonergodic stochastic processes. After reading my 1982–83 article on the fallacy of rational expectations, Nobel laureate Sir John Hicks wrote me a letter dated February 12, 1983, in which he stated: “I have just been reading your RE (rational expectations) paper … I do like it very much…. You have now rationalized my suspicions and shown me I missed a chance of labeling my own view as nonergodic. One needs a name like that to ram a point home.” In a letter dated May 21, 1985, Noble laureate Robert M. Solow wrote me, “Let me first say that I always admired that article of yours on nonergodic processes and I thought it was right on the button.” Furthermore, Nobel laureate Douglass North (2005, p. 19) explicitly cites my emphasis on nonergodic analysis with changes in future economic events in his 2005 book to help readers on *Understanding the Process of Economic Change*.

The ergodic axiom is fundamental to the theory of stochastic processes, which is, in essence, the modern twentieth-century theory of probability. The origin of this ergodic axiom concept, it is claimed, can be traced back to eighteen- and nineteenth-century statisticians such as Bernoulli, Maxwell, Boltzmann, and Poincaré. In 1898, Boltzmann coined the word “ergode” in his
discussion of the analysis of Poincaré. In a German-language paper published in 1912, Paul and Tatiana Ehrenfest formulated the “ergodic hypothesis,” which is the basis of calculating probabilities from existing historical data to make actuarial predictions about future outcomes. Paul Ehrenfest was the major professor of Tinbergen, and Ehrenfest was influential in having his ideas regarding ergodicity applied by Tinbergen to economics. Consequently, Keynes’s (1939) attack on Professor Tinbergen’s method can be interpreted as a rejection of the ergodic axiom.

If one conceives of the path of any economic system’s events over time into the future as governed by a stochastic (probability) generating process, then an event can be accurately forecasted in terms of a probability distribution. Logically speaking, to determine the characteristics of any future event one should draw and statistically analyze a sample from this future. Since it is impossible to draw a sample from the future, the concept of ergodicity permits the analyst to assert that any sample drawn from past data can be analyzed to provide a probability distribution that is equivalent to the probability distribution that would be drawn from the future.

Accordingly, the ergodic axiom indicates that a universal truth is that it is scientifically possible today to calculate a probability distribution applicable to future outcomes. Analyzing a sample of existing data-facts will provide the basis for a computation that provides an actuarial certainty regarding any future outcome. There can be no uncertainty regarding the future.

As we will show below, Keynes can be specifically quoted to indicate that this type of ergodic probability analysis results in fallacies. Keynes’s concept of uncertainty requires the rejection of the ergodic axiom as a basis for his more general theory than that of the twentieth-century classical theory of the operation of our economic system.

Keynes’ separation of uncertainty from probable events was developed in his earlier work *A Treatise on Probability* (1921), where in chapter 30 Keynes strongly attacked the LaPlacian school (the forerunners of modern Bayesian probability analysis). This attack on the use of probability analysis is carried on in the

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1 I am grateful to an unnamed referee who provided this information regarding the history of the development of ergodic stochastic theory and the reference to Jos Uffink’s demonstration that Keynes’s separation of uncertainty from probability had an effect on those who were studying ergodicity.
Although Keynes never used the term *ergodic* in his writings on uncertainty, an anonymous referee for this rejoinder indicated that Uffink (2006) demonstrated that Keynes’s *Treatise on Probability* had a significant influence on discussions of those studying ergodicity. Apparently, even if Keynes was not exposed to the theory of ergodic stochastic processes, his work on uncertainty and the use of probability analysis was sufficient to have important implications for those studying ergodicity.

If Keynes, in his lifetime, was never directly exposed to the lexicon of the then developing stochastic theory, then it easy to understand why he did not explicitly stated that his general theory required the rejection of the ergodic axiom. Instead he merely argued against using probability analysis to forecast the future and stated there is no basis for developing a scientific calculation of an actuarial certain knowledge of future outcomes. In his 1937 article in the *Quarterly Journal of Economics* explaining the basis for his *General Theory*, the classical assumption Keynes believes must be overthrown in permitting uncertainty to affect entrepreneurial decision making is specified. Keynes explicitly stated that in classical theory:

> **Facts and expectations were assumed to be given in a definite form; and risks… were supposed to be capable of an *exact actuarial computation*. The calculus of probability… were supposed capable of reducing uncertainty to the same calculable state of certainty itself…. I accuse the classical economic theory of being itself one of these pretty, polite tech-niques which tries to deal with the present by abstracting from the fact we know very little about the future…. [The classical economist] has overlooked the precise nature of the difference which his abstraction makes between theory and practice, and the character of the fallacies into which he is likely to be lead. (Keynes, 1937, pp. 112–115; emphasis added)**

In this statement, Keynes explicitly indicated that his general theory required him to reject the twentieth-century classical theory’s assumption that decision makers use probabilities to calculate

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2 In the *General Theory*, Keynes (1936, p. 152) continued this attack on the use of probability theory when he wrote “Nor can we rationalise our behavior by arguing that to a man in a state of ignorance errors in either direction are equally probable, so that there remains a mean actuarial expectation based on equi-probabilities. For it can easily be shown that the assumption of arithmetically equal probabilities based on a state of ignorance leads to absurdities.”
an actuarially certain knowledge of future events. Thus O’Donnell is incorrect when he indicates that Keynes only rejected “the perfect knowledge axiom.”

I have written (e.g., Davidson, 2011, p. 35) that the early nineteenth-century economists such as Ricardo assumed that every decision maker had perfect knowledge about the future. If decision makers have perfect knowledge about the future they do not need to use probabilities to develop actuarial knowledge. By the beginning of the twentieth century, classical mainstream theory had abandoned the perfect knowledge axiom. Instead these theorists were identifying knowledge about the future with probabilistic risk in that the latter could be calculated from existing data. In other words this conversion of risk calculations to “actuarial” certainty knowledge about the future required the classical economic theory of Keynes’s time to implicitly assume that the economy was generated by what modern probability (stochastic) theory calls an ergodic stochastic process.

Keynes’s 1936 rejection of the classical theory’s belief that decision makers had any knowledge about the future requires Keynes not only to abandon the “perfect knowledge” axiom of the nineteenth-century classical theorists, but also the twentieth-century classical axiom that the future could be reduced to a computed actuarial certain knowledge.

Or as I have often written (e.g., Davidson, 2011, pp. 34–38), in today’s deterministic models (i.e., the equivalent of nineteenth-century classical models), Keynes’s concept of uncertainty requires overthrowing the ordering axiom that presumes that each decision maker has perfect knowledge of the future payouts (value) associated with all possible alternative decisions that can be made today. In probabilistic (stochastic) models, Keynes’s uncertainty concept requires throwing over the ergodic axiom that presumes the future can be reduced to an actuarial computed certainty.

O’Donnell’s epistemological approach requires him to admit he does not know (understand?) whether real world economic data are being generated by any ergodic or nonergodic stochastic process or not. O’Donnell states he has to wait until infinity to determine whether the stochastic process that generates economic data-facts is ergodic. But if this is the best defense O’Donnell can provide for his epistemological argument, then he does not really understand the definition of an axiom and its relevance for creating any general theory. If any axiom is used as the basis for
developing a theory, then the theorist who invokes this theory is implicitly claiming that that underlying axiom is a statement that he/she believes must be universally accepted as true. An axiom is a statement that needs no proof.

For Keynes to develop a more general theory than twentieth-century classical theory, he had to reject the ergodic axiom that restricts the theory to an explanation of an economic system where decision makers could know the future through an actuarial calculation. In this twentieth-century classical theory where it is presumed that decision makers used probability analysis to obtain an actuarial certain future, Keynes’s concept of uncertainty cannot exist.

Keynes argued that the mainstream classical theorists of his time “resemble Euclidean geometers in a non-Euclidean world who, discovering that in experience straight lines apparently parallel often meet, rebuke the lines for not keeping straight…. Yet, in truth there is no remedy except to throw over the axiom of parallels and to work out a non-Euclidean geometry. Something similar is required today in economics” (Keynes, 1936, p. 16, emphasis added). In other words, Keynes argued that economists had to “throw over” some fundamental axioms that are the foundation of twentieth-century classical theory to produce a theory able to explain why persistent unemployment and the use of money exists in our economic world of experience.

Furthermore, in the Preface to the 1936 German-language edition of the General Theory, the following sentences appear: “One of the reasons which justify my calling my theory a General Theory. Since it is based on fewer restrictive assumptions (weniger enge Vorasussetzungen Stutz) than the orthodox theory, it is also more easily adopted to a large area of different circumstances.”

In other words, Keynes claimed that the fewer restrictive axioms used to formulate a theory, the more general the theory was to be applicable to different circumstances. Keynes makes the concept of uncertainty about future economic events a focal point of his general theory. Yet the restrictive ergodic axiom involving probability analysis used by the classical theorists of his time insists that the future is never uncertain because decision makers can use the existing data to calculate an actuarial certain knowledge of the future.

*These sentences do not appear in The Collected Writings of John Maynard Keynes, vol. 17. B. Schefold (1980, pp. 175–176) has called attention to the fact that these sentences do not appear in The Collected Writings.*
Accordingly, if one is to develop a general theory that can explain what occurs when the future is truly uncertain even to ignorant humans, then Keynes had to “throw over” the classical theory’s ergodic (probability) axiom to explain what can occur when decision makers know that they cannot know the future, that is, decision makers recognize that there is an uncertain economic future that cannot be reduced to an actuarial certain calculation.

Logic dictates that the onus is on those who insist on adding additional restrictive axioms to a general theory to justify the use of these additional axioms. Thus, if the classical theorists of Keynes’s time insisted it was a universal truth that decision makers can make actuarial certain forecasts regarding the future (i.e., the ergodic axiom), then logically they are required to justify the use of that axiom. Since Keynes invoked a more general theory than the classical theory, Keynes and his post Keynesian followers are not required to prove whether the actual historical economic data are not generated by an ergodic process. In trying to explain the operations of the money-using, market-oriented economic system where the future is uncertain and there is no scientific basis for calculating any actuarial certain knowledge, all that Keynes required is to “throw over” the assumption that one can calculate probabilities today from existing data to create an actuarial certain future.

Yet O’Donnell declares that my ergodic/nonergodic approach “is alien to Keynes’s thought, ignores his actual analysis of uncertainty and probability in the TP [Treatise on Probability] and its links to the GT and falsely claims his implicit acceptance of the ergodic/nonergodic distinction…. The basic axiom that Keynes abandoned in the GT was not the ergodic axiom, but the perfect knowledge axiom” (O’Donnell, 2014–15, p. 207, emphasis added).

O’Donnell claims that Keynes overthrew only the absolute certainty assumption—(the ordering axiom) —and implicitly denies that Keynes knew about and rejected the twentieth-century classical theorists’ assumption that decision makers could calculate “actuarial” knowledge of the future. Despite his claim that ergodic/nonergodic proponents’ treatment of Keynes’s works is “inaccurate,” O’Donnell exhibits no knowledge of Keynes’s 1937 Quarterly Journal of Economics essay explaining that his general understanding.

4 Even if ignorant due to some epistemological view of O’Donnell, Bayesian probability theorists insist these ignorant humans will calculate an actuarial expectation based on equi-probabilities (see note 2).
theory rejects the classical assumption that the future can be determined in terms of probabilities into an actuarial certainty.

In this 1937 explanation of the *General Theory*, Keynes is specifically attacking the twentieth-century classical theory view that decision makers can make “an exact actuarial computation” of future events that reduces uncertainty to a “state of certainty.” Since the classical theorists of his day had already given up the perfect knowledge axiom (the ordering axiom) of Ricardo, Keynes was not foolish enough to simply beat the dead horse classical axiom of perfect knowledge. Rather in his 1937 Quarterly Journal of Economics article it is clear that Keynes is rejecting the classical theorists’ assumption that the future could be actuarially calculated (known) via probability theory analysis. Consequently, it is O’Donnell who is “inaccurate” when he states that the classical absolute certainty assumption is the *only* basic axiom that Keynes’s *General Theory* throws over. O’Donnell apparently does not perceive the difference between the ordering axiom and the ergodic axiom.

Keynes stated:

By “uncertain” knowledge… I do not mean merely to distinguish what is known for certain from what is probable…. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention…. About these matters there is no scientific basis on which to form any calculable probability whatever. (Keynes, 1937, pp. 114–115)

Since the classical economists of Keynes’s time were using probability analysis to calculate an actuarial certain future, then they must have assumed the economy was being generated by an ergodic stochastic process even if they did not use the stochastic (probability) theory terminology. In this stochastic (probability) analysis, there can be *no scientific basis* for calculating actuarial knowledge regarding future economic events from existing economic facts and data if the *stochastic process generating the facts and data is nonergodic*. Consequently, Keynes’s concept of uncertainty involving a system where he specifically states there is “no scientific basis” for producing an actuarial certain computation regarding future economic events is logically consistent with the stochastic theory
classification implication that economic facts are generated by a nonergodic stochastic process.

Keynes’s general theory does not require assuming humans have no epistemological knowledge of how to scientifically compute and calculate statistically significant predictions of future events in areas where the facts are generated by ergodic stochastic (probability) systems, such as in the insurance industry and many hard sciences. As the anonymous referee of this rejoinder has stated, in the Treatise on Probability Keynes “does raise the issue of insurability and recognizes that there are cases where such probabilities are known.” Accordingly, Keynes recognized that it was not epistemological uncertainty but rather ontological uncertainty that apparently separated insurable (predictable) future events from uncertain future economic events.

Reasoning requires the human mind to create a theory to explain what people observe happening around them. Axioms are the foundation on which the theorist uses the laws of logic to reach conclusions to explain what we observe in the world of experience. Thus if non-Euclideans must throw over the axiom of parallels in their theory of geometry, then there is an alternative they need not prove, even if they can, but must accept. This is that if there are straight lines in the system that, at any point, appear to be parallel, then these straight lines may approach (converge to) a point where they meet or may even diverge.5

In modern stochastic (probability) theory language then, Keynes’s claim that the existing economic data cannot be used to calculate the actuarial price of copper twenty years hence, and so on, means that if economic theory is to be relevant to Keynes’s view of the real world of experience, then his General Theory must overthrow the ergodic axiom. If using the ergodic axiom leads to assuming that people can actuarially know the future when Keynes argues we cannot know the future and the classical theory leads to many “fallacies,” then the only logical alternative that Keynes’s economic theory must permit (without having to prove it) is that existing economic data (facts) may be generated by a nonergodic stochastic process. If the data are generated by a nonergodic system, then there is no scientific method that permits the future to be actuarially computed from any existing database. The future

5 Thus longitudinal lines on a global model of the Earth that are parallel at the equator will meet at the north and south poles of the globe.
must remain uncertain no matter how much historical data are collected and analyzed.

On the other hand, O’Donnell wants us to believe that even if in 1937 Keynes explicitly rejected the actuarial computation probabilistic approach to knowing the future, Keynes did not, and could not, suggest whether the stochastic process that generates economic facts is ergodic or not. O’Donnell’s epistemological approach suggests that neither Keynes nor any mortals today can ever know—until we reach infinity in time—whether the economic data are generated by either an ergodic process or a nonergodic process. Yet there are mathematical and statistical criteria for finding out whether a series of data are generated by an ergodic or a nonergodic process without waiting for infinity (e.g., Malinvaud, 1966, pp. 387–388).

Nevertheless, Keynes specifically indicated his view that the assumption that an actuarial computed certain economic future can be calculated from the existing “facts” (i.e., the ergodic axiom) results in “fallacies.” Logically then Keynes’s theory must be open to the alternative that in terms of modern probability theory the existing economic data may be generated by a nonergodic process. Moreover, users of Keynes’s general theory are not required in logic to prove that the rejected ergodic axiom is not a necessary axiom to explain the world of experience. So, despite O’Donnell’s claim to the contrary, Keynes and post Keynesians do not have to wait until infinity to see if certain economic data are generated by an ergodic system before explaining how uncertainty in a nonergodic system affects the operation of the money-using, market-oriented capitalist economy in which we live.

O’Donnell ultimately rests his “critique” on the definition of an ergodic process where I noted that the time averages and space (cross-sectional) ensemble averages will converge—with the probability of unity—with the time average equaling the space average at infinity. O’Donnell jumps on this definition to declare that until humans reach infinity time any concept about decision makers’ uncertainty about the future must be epistemological and not ontologically based. O’Donnell insists that only at infinity can a decision maker know for certain whether the time statistics and the space statistics will meet and therefore whether the system is ergodic or not.

There are several problems with this aspect of O’Donnell’s critique. The most important problem is that apparently
O’Donnell does not realize that since Keynes rejected the classical assumption that it was “scientifically” possible for economic decision makers to calculate “actuarial” future economic events, then whatever is generating the economic data, according to modern probability theory, must be a nonergodic stochastic process. Keynes and the post Keynesians do not have to prove that the system is nonergodic. Nor is it necessary to prove that human decision makers, or even O’Donnell, have an epistemological lack of knowledge about whether a nonergodic stochastic process generates economic data (“facts”). As long as Keynes’s general theory rejects the restrictive classical ergodic axiom that presumes it is possible for decision makers to scientifically calculate actuarial values of future economic events from existing data, then the general theory must allow that economic data can be generated by a nonergodic stochastic process and therefore there is no “scientific method” using probability theory by which decision makers can calculate actuarial values for some future economic events!

Nevertheless, O’Donnell refuses to recognize that the onus is not on those who reject a restrictive ergodic axiom to prove this axiom is not necessary for their more general theory of the operation of the economy. Instead, O’Donnell launches into a tirade about whether humans can prove (know) that the economic system is ergodic or not when he declares that “First the paths taken by the series, both within each (time and space statistics) series and between them, are completely unstipulated, which means they can take any conceivable form so long as the distributional identity emerges at infinity” (O’Donnell, 2014–15, p. 192). O’Donnell then states:

Path indeterminacy again prevents this. The convergence/nonconvergence that matters occurs at infinity, a never-arriving destination in time and space. With unspecified convergence processes, the preinfinity signals emitted by reality are unconstrained…. Furthermore, convergence is only a “tendency” and tendencies arise when underlying forces are opposed by countervailing forces. Whether the former or the latter currently dominate emitted signals, and over what time and space is unobservable, so that again nothing useful can be inferred from observations of convergence/nonconvergence over trivially short spans of time or space. Prior to infinity there can be no necessary or sufficient

All this path discussion is irrelevant to the ergodic/nonergodic analysis since in his 1937 Quarterly Journal of Economics article, Keynes specifically rejected the classical assumption that it is possible to actuarially compute future economic events which in modern probability theory involves assuming that the data are generated by an ergodic stochastic process. The resulting general theory must permit the existence of uncertainty may be due to data generated by a nonergodic system.

O’Donnell engages in furthering his irrelevant argument by stating that for less than infinity time the path of the statistics over time may actually continually move farther apart even if the system is always ergodic. O’Donnell writes: “Over finite time and space the two distributions may appear to be (a) converging and hence indicating ergodicity when in fact they are generated by nonergodicity, or (b) diverging and hence suggesting nonergodicity when in fact the underlying reality is ergodicity” (O’Donnell, 2014–15, p. 192).

Whether O’Donnell’s statements that paths of statistics before infinity are “completely unstipulated, which means they can take any conceivable form so long as the distributional identity emerges at infinity” and “the preinfinity signals emitted by reality are unconstrained” are consistent or not with the properties associated with the convergence of ergodic stochastic distributions are not relevant for a discussion of the basis of Keynes’s general theory involving uncertainty about the future. O’Donnell’s lengthy discussion of paths and waiting for infinity to decide on ergodicity has no relevance since Keynes and Post Keynesians summarily reject the ergodic axiom as part of their general theory. Furthermore, a nonergodic stochastic process is completely consistent with Keynes’s explicit statement that there is no scientific way of making correct actuarial calculations regarding future events.

Even though O’Donnell’s remarks about the paths of the time and space statistics before infinity are completely irrelevant, it may be of interest to some readers to find a complete mathematical proof that the concept of convergence of stochastic processes is not the system described by O’Donnell (2014–15, p. 192) where the time statistic and the path statistic distributions over “finite time…
may appear to be ... diverging and hence suggesting nonergodicity when in fact the underlying reality is ergodic.”

If the system is ergodic then at any point of time before infinity the time statistical average and the space (cross-sectional) statistical average may not be equal due to sampling errors. Nevertheless, if the system is ergodic, the sampling error becomes relatively smaller as the number of sample observations increases over time. Accordingly, as the sampling error becomes relatively smaller over time, the space statistics path and the time statistics path should be converging, that is, moving closer to each other over time.

O’Donnell (2014–15, p. 206,) also claims that “In general terms, the critique of the ENE [ergodic/nonergodic] framework presented here is applicable to all theories.” If he really means his critique is applicable to all theories, then astronomers, physicists, and other hard scientists can never know whether they are dealing with an ergodic system and hence they cannot know if they can make statistically reliable forecasts about what will happen at future dates. If O’Donnell’s epistemological argument is applicable to all theories, astronomers should not be able to use their theory to predict when the next solar eclipse will occur—because, according to O’Donnell everything that happens in our world is clouded by an epistemological uncertainty of human knowledge that exists since we never get to infinity! Yet astronomers have an excellent record of being able to predict to the minute when and where the next solar eclipse will occur. Are these predictions just lucky calls?

Moreover O’Donnell argues that over the short period of time that we humans live and collect data that “we cannot impose any (preferred) forms on how it undertakes convergence or nonconvergence” and that “prior to infinity, there can be no necessary or sufficient condition for ergodicity/nonergodicity.” I reiterate that this statement about imposing forms is irrelevant for the discussion of whether Keynes’s general theory allows data to be generated by a nonergodic process. But it should be noted that a number of stochastic theorists have indicated why ergodic theory is necessary to forecast future outcomes. For example, Billingsley (1978, p. 1) states that if “the laws governing ... change remain fixed as time passes ... [then] ergodic theory is a key to understanding these fluctuations.” Billingsley

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6 This mathematical proof is available at http://smat.epfl.ch/courses/theory/week2.ht.pdf.
(1978, p. 2) goes on to state that whenever the “passage of time does not affect the set of joint probability laws governing experimentation (outcomes), then the assumption of ergodicity permits regularities to be perceived from what might at first sight be patternless fluctuations.” In other words, assuming ergodicity permits past pattern of regularities to be reliably projected into the future.

O’Donnell (2014–15, p. 205) argues, that Keynes’s criticism of Tinbergen’s method was derived from Lexis and Bortkiewicz, “and does not represent any implicit reaching out to ideas of later vintage.” Yet, the more precise taxonomy of modern stochastic theory over the 1930s probability theory lexicon would certainly be one Keynes would have readily adopted to help him explain the role of uncertainty in his General Theory. Keynes wrote that philosopher G.E. Moore’s book Principia Ethica greatly affected him. Keynes (1949, p. 88) wrote, “Under the influence of Moore’s method … you could hope to make essentially vague notions clear by using a precise language about them and asking exact questions.” Stochastic theory provides a precise language to distinguish between two classes of “facts”: (1) data systems that permit actuarial certainty (an ergodic system) calculations, and (2) data systems where no scientific actuarial certainty (a nonergodic system) about the future can be obtained.

Harrod, Keynes’s first biographer, indicated that it was this drive for a precise taxonomy and exposition that permitted Keynes to break the grip of the classical economics that he had been exposed to at Cambridge. Harrod (1951, pp. 463–464) states: “It is true to say that the Keynesian scheme consisted in essence in a new set of definitions and classification …. Classification in economics, as in biology, is crucial to scientific structure.”

This clarity of a precise taxonomy and classification is what Hicks obviously recognized in his letter to me (cited above) when he wrote, “I missed a chance of labeling my own view as nonergodic. One needs a name like that to ram a point home.”

Conclusion

There are additional errors in O’Donnell’s critique. For example, O’Donnell’s claim that the basis of Knight’s distinction between risk and uncertainty can “fit the ENE framework” (O’Donnell,
2014–15 pp. 190–191). By using direct quotes from Knight, I have demonstrated elsewhere (Davidson, 2007, pp. 106–107) that Knight’s concept of uncertainty is not the same as Keynes’s and Knight’s distinction does not “fit the ENE analysis.”

In fact O’Donnell’s epistemological approach to uncertainty is simply a variant of Knight’s conception of uncertainty as reflecting an epistemological problem. It should be noted that although Knight was at the University of Chicago, his uncertainty argument had absolutely no effect on, and is consistent with, the Chicago School’s laissez-faire analysis arguing that the market knows better than individuals as to future outcomes and therefore government should not intervene or try to regulate free markets. In a world of epistemological uncertainty, policymakers have no more understanding of the economic future than individuals in the market. In such a world, only the fittest decision makers in the market will survive, resulting in a Darwinist-evolving stronger economic community!

Knight (1921, p. 210) writes that the “universe may not be knowable…but objective phenomenon is certainly knowable to a degree so far beyond our actual powers…any limitation of knowledge due to lack of real consistency [i.e., lack of ergodicity] in the cosmos may be ignored.” In other words, Knight is adopting an epistemological concept of uncertainty. Furthermore, Knight states that it “is conceivable that all changes might take place in accordance with known laws” (1921, p. 198). Accordingly, Knight recognizes the possibility that (1) the economy is ergodic (i.e., does have real constancy over time), and/or (2) future economic events may be compatible with economic changes in the future taking place under known natural laws. Consequently, for Knight uncertainty is an epistemological concept that exists because humans may not have the “power” to know when future events are predetermined by known laws controlling economic activity.

O’Donnell’s epistemological view of uncertainty is completely compatible with Knight’s views. The economic universe may have consistency over time (the system is ergodic) even if O’Donnell cannot know this until infinity in time is reached. If, as O’Donnell declares, no humans (whether policymakers or mere individuals) can ever possess knowledge about the possible ergodic economic system, then why should economic policies and government regulation of markets—including financial markets—produce a better
economic future? Instead, if the system is ergodic then if we leave it to the market to find the real consistency of possible changes in the economic universe over time that is already built in via economic laws that govern economic outcomes, will the future not be as good as possible? Or as Ronald Reagan famously asked: “Why should some bureaucrats in Washington know better than you how to spend your income?”

For Knight:

The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known (either through calculation *a priori* or from statistics of past experience), while in the case of uncertainty, this is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique. (Knight, 1921, p. 233)

Note that many defenders of free financial markets when asked why they did not see the global financial collapse of 2007–8 coming replied that such an occurrence would happen only once in 5,000 years—that is, it was highly unique. So government regulations could not have seen this coming (even if the Glass–Steagall Act had not been repealed during the Clinton administration?).

In sum, then, the epistemological uncertainty approach whether presented by Knight or O’Donnell provides free market advocates the basis to claim that no one (whether policymaker, individual, Keynes, or Post Keynesian) knows what the future will bring. Accordingly, it is better to let individuals in the marketplace make their own decisions and errors. A government policy of laissez faire will result in survival of the fittest. Those who make the fewest errors will survive and hence a stronger economic system in an epistemological uncertain economic world will be developed.

There is no room for this epistemological approach to uncertainty laissez-faire philosophy in a Post Keynesian analysis that is derived from Keynes’s *General Theory*. This is because Keynes (1936, p. 372) noted that the two “outstanding faults of the economic society in which we live are its failure to provide for full employment and its arbitrary and inequitable distribution of wealth and income. The bearing of the foregoing (general) theory on the first of these is obvious.” Keynes insisted that when unemployment occurs, it is up to government to actively adopt policies
that will immediately increase aggregate market demand for goods and services. Keynes would never have accepted the implications of the Knight–O’Donnell epistemological view that provides support for a laissez-faire approach to relieving the two outstanding faults of the capitalist economy in which we live.

I see no reason to provide a complete index of all the other potential errors or misrepresentations in O’Donnell’s critique of the ergodic/nonergodic approach. I am not interested in presenting a diatribe against O’Donnell as a scholar. I can only hope that with the information in this rejoinder, I can convince the objective reader that the ergodic/nonergodic approach is the modern probability (stochastic) basis for Keynes’s general theory analysis of an economy with an uncertain future, and that this approach provides useful ways of explaining the operations of a market-oriented, money-using, and capitalist system.

REFERENCES